Application of the inversion method to a real-time far-field tsunami forecast system

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Abstract. We applied a new inversion method using wavelet transform to a part of the real-time tsunami forecast system, which is for estimating the 2-D initial waveform. Since this inversion method doesn't require fault location, we can analyze a tsunami in real time without any seismic information. In order to study the reliability of the method, a wavelet inversion method combined with a numerical simulation was applied to a hypothetical tsunami source off the Taiwan Sea. The correlation coefficient for the estimated initial waveform was calculated to be 0.78, suggesting high reliability of the method. The sum of the tsunami arrival time at the tsunami meters used in this study (45 min), the observed time (45 min), and the analyzing time for inversion and forward propagations are estimated to be about 95 min in this case area, indicating that once the location of tsunami gauges are installed, we can know the necessary time to forecast tsunami heights, adding the observed time and the analyzing time to the arrival time. Comparing the tsunami heights forecast by this method with those calculated by the fault model shows that the average error of wave heights and arrival time are 0.39 m and 0.007 min, respectively.

1. Introduction

In order to obtain reasonable results from the numerical simulation of a tsunami, three important factors—accurate initial conditions, proper governing equations, and appropriate mesh size—are required. Previous research (e.g., Imamura and Goto, 1988) focused on the governing equations and numerical errors inevitably induced in the simulation. Some criteria to select equations and mesh sizes have already been proposed (e.g., Shuto *et al.*, 1990). However, another problem is the tsunami initial waveform, which is used as the initial condition of the numerical computation. Although the initial condition can be calculated from fault parameters using Mansinha and Smylie (1971), information from seismic waves is not enough to determine all of the parameters in a short time.

Six fault parameters and fault location are used to calculate the deformation of sea bottom, which in turn yields initial tsunami waveform. Three parameters can be estimated from the CMT solution method (e.g., Dziewonski *et al.*, 1981). The magnitude or energy of the earthquake can give a relationship among the other three parameters—length, width, and dislocation. More information to estimate them is necessary. That information is an aftershock area, corresponding to the area of a tsunami source. The location and size of a tsunami source are commonly assumed to be given by the aftershock data. However, it often takes several days to obtain enough aftershock

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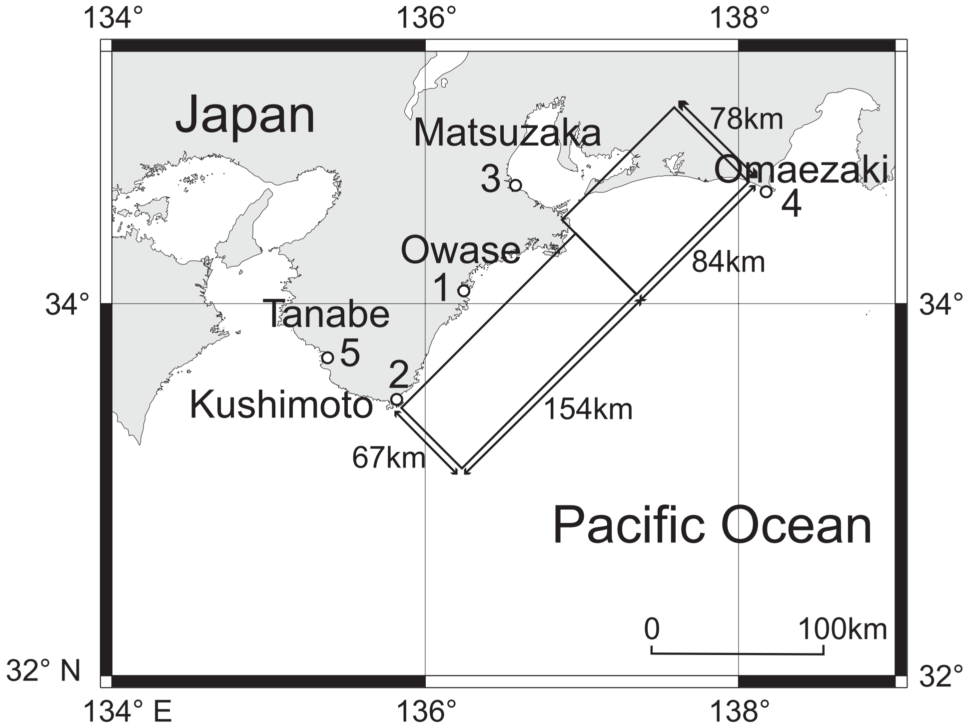

Figure 1: Location of five tidal stations and the 1944 Tonankai earthquake fault model proposed by Aida (1979).

data to estimate the source area. For real-time analysis, a location method not using aftershock data is needed.

Another way to determine the fault parameters is to use tsunami waveform data. Aida (1972) proposed the inversion method to estimate the dislocation on a fault using tsunami waveforms observed at tidal stations and their Green function. Satake (1987) improved the method to evaluate heterogeneity of dislocation on a fault using tsunami waveforms at more stations. Such inversion methods revealed that many earthquakes have a complex distribution of dislocation. However, all methods assumed the location of the source, which is generally unknown in real-time analysis.

In this study, a new inversion method to estimate the location and water disturbance using the wavelet transform is proposed. Since periodic functions are not necessary in the wavelet transform, it is possible to estimate initial distribution of tsunamis caused not only by earthquakes but also landslides or volcanos, which have no aftershocks.

Since the present forecast system uses only the magnitude of an earthquake or a database of tsunami heights, a real-time forecast system still does not exist. Since the arrival time is more than a few hours in a far-field tsunami, there is available time to forecast in real time the tsunami heights using time-series water level data at tsunami gauges.

2. Tsunami Inversion Method

Since the governing equations of tsunamis in water deeper than 50 m are considered linear, the principle of superposition can be used in solving the equations. A tsunami initial waveform $\mathbf{e}_0$ can be decomposed into some bases of 2-D vector $\mathbf{e}_k (k = 1, 2, \cdots, n)$ as follows:

$$\mathbf{e}_0 = c_1\mathbf{e}_1 + c_2\mathbf{e}_2 + \cdots + c_n\mathbf{e}_n \tag{1}$$

Applying this expansion and the principle of superposition to the numerical computation of tsunamis yields the following equation:

$$\eta(t) = c_1\eta_1(t) + c_2\eta_2(t) + \cdots + c_n\eta_n(t) \tag{2}$$

where $\eta(t)$ is the computed waveform time series at a tidal station. $\eta_k(t)(k = 1, 2, \cdots, n)$ is the computed waveform time series when each basis in (1) is regarded as the initial condition of numerical computation. c_k is the same as in (1). In this study, the wavelet of Beylkin (Beylkin *et al.*, 1991) is used as $\mathbf{e}_k$ in (1), because it is the best for solving the inverse problem among three wavelets, i.e., Beylkin, Daubechies and Coiflet.

An inversion method to estimate a tsunami initial waveform from tidal station data is proposed as follows. Regarding $\eta(t)$ in (2) as the observed data and $c_k(k = 1, 2, \cdots, n)$ in (2) as unknown variables, the relationship between the initial waveform and a later time series in the inversion method is expressed by

$$\mathbf{A}_{ij}(t) \cdot \mathbf{x}_j = \mathbf{b}_i(t) \tag{3}$$

where $\mathbf{A}_{ij}(t)$ is the computed time series of waveforms, or Green's functions, at the ith station from the jth wavelet basis, $\mathbf{x}_j$ is the coefficient of the jth wavelet basis, and $\mathbf{b}_i(t)$ is the observed tsunami waveform at the ith station. The coefficient $\mathbf{x}_j$ of each wavelet basis can be calculated by a least-squares method. Thus, a tsunami initial waveform can be estimated by substituting these $\mathbf{x}_j$ for c_j in (1).

A water disturbance caused by any geophysical phenomena can be estimated by the above procedure, whereas previous methods by Aida (1972) and Satake (1987) give only dislocation and water displacement, and are, therefore, limited to fault-generated tsunamis.

3. Results

We apply the new inversion method to the 1944 Tonankai tsunami. The waveform time series at the five stations shown in Fig. 1 are artificially made by the numerical simulation of a tsunami, because they are not available. The tsunami simulation has a computational domain of 400×400 nodes with spatial grid size, Δx, of 1 km. The new inversion method is applied using the computed waveforms at each tidal station and a domain of 256×256 nodes (Δx = 1 km), as shown in Fig. 2. The computational area was reduced to save CPU time during wavelet transformation. In estimating the initial

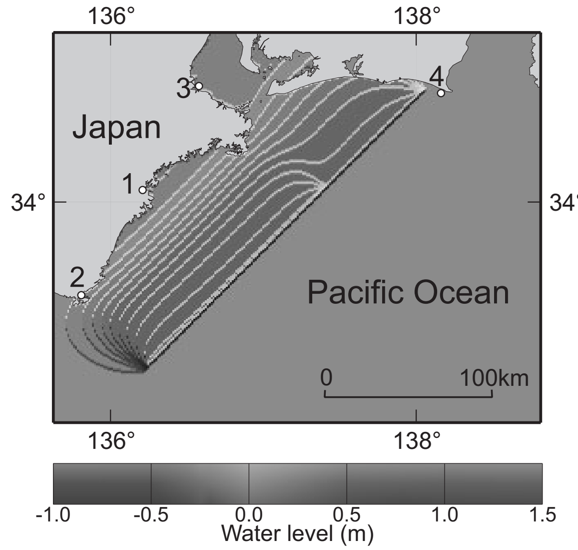

Figure 2: Tsunami initial waveform calculated by the fault parameters of the 1944 Tonankai earthquake (Aida, 1979).

waveform using (1), only 64 wavelet bases from the total of 65,536 bases were used in order.

In order to evaluate validity of the new method, we define a correlation coefficient as

$$\frac{E\left[(\eta_{\mathrm{tru}}(i,j)-\mu_{\mathrm{tru}})(\eta_{\mathrm{est}}(i,j)-\mu_{\mathrm{est}})\right]}{\sqrt{E\left[(\eta_{\mathrm{tru}}(i,j)-\mu_{\mathrm{tru}})^2\right]E\left[(\eta_{\mathrm{est}}(i,j)-\mu_{\mathrm{est}})^2\right]}}$$

where E is expectation. $\eta(i,j)$ is the water displacement at calculation node (i,j). μ is the expectation of $\eta(i,j)$. The subscript “tru” stands for “truth” and the subscript “est” stands for “estimation.”

Table 1 shows resulting correlation coefficients and average errors of the estimated initial forms for each case. Ideally, this method's accuracy should depend only on the number of waveform time series at each station. We can understand from the table that the more waveforms used at each station, the better the result. However, the addition of waveforms from far-field locations or unaffected areas does not improve the result, as shown in case 4 of Table 1. From this study, waveforms from four stations are recommended

Table 1: Correlation coefficient of the estimated waveform and the calculated waveform.

Tidal station number	Correlation coefficient
2	0.70
3	0.79
4	0.85
5	0.76

to be used in the new method, because the correlation coefficient and the average error of case 3 are the best, as shown in Table 1.

The previous method of Satake (1989) decomposed a fault into several subfaults and estimated the distributed slip on each subfault. The eight subfaults and their locations using Satake's method are assumed, as shown in Fig. 3. The amount of slip on each subfault was estimated using Satake's method and the initial form is calculated assuming the fault mechanism and using the Mansinha and Smylie (1971) method. The correlation coefficient for this method is 0.82. The results of the new inversion method are compared to those of the previous method. Table 1 shows that the correlation coefficient for the new method is 0.85, indicating that both methods give results with about the same accuracy. However, the source area and the fault mechanism do not have to be assumed in the new method. This is of great advantage to estimate a tsunami source area in time when the seismic information is not yet obtained and to study the source of aseismic tsunamis caused by landslides or volcanoes.

4. Application of the Inversion Method to a Real-Time Forecast System

The process of applying this inversion method to a real-time forecast system is as follows: first, water level data are recorded at tsunami gauges. Second, using these water level data, tsunami initial waveform is estimated using the inversion method. Finally, regarding this tsunami initial waveform as the initial condition of numerical computations, the tsunami heights along coasts are calculated by numerical computations.

In order to check the usability of this system, a numerical simulation was executed assuming an earthquake at sea off Taiwan. Because Taiwan is the nearest country from Japan and the traveling time of tsunamis is the shortest, this area is adopted as the computational area. Figure 4 shows the location of the fault model assumed at sea off Taiwan and two tsunami gauges. The computational area is a 256×256 mesh area, and the grid size is 3 km. Only 64 wavelet bases are used when the inversion method is executed.

The left panel in Fig. 5 shows the tsunami initial waveform calculated by the fault model using the Mansinha and Smylie (1971) method. The right panel figure shows the tsunami initial waveform estimated by the inversion method using water level data observed at two tsunami gauges. The correlation coefficient for the estimated initial waveform to the assumed one was calculated to be 0.78.

Figure 6 shows time-series waveform data at the tsunami gauge at 1 in Fig. 7. It takes about 45 minutes for a tsunami to arrive at the tsunami gauge. From that time, the waveform at point 1 in Fig. 7 are observed for about 45 minutes to be analyzed. It takes a total of 90 minutes to observe time-series waveform data from tsunami gauges for the inversion method. After that, it takes 5 seconds to estimate the 2-D initial waveform using the inversion method. Moreover, it takes 2 minutes to simulate the

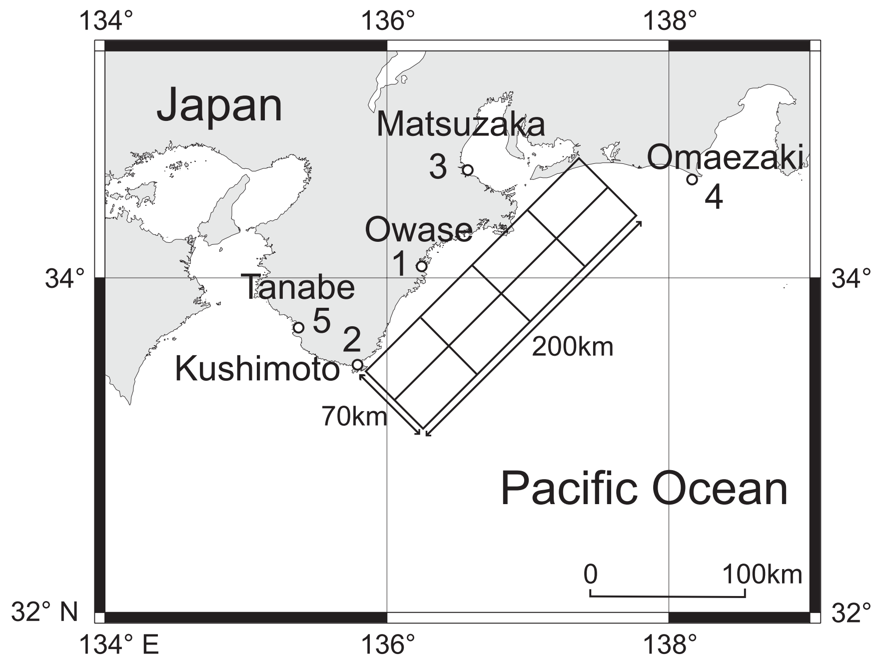

Figure 3: Assumed fault model divided into eight subfaults.

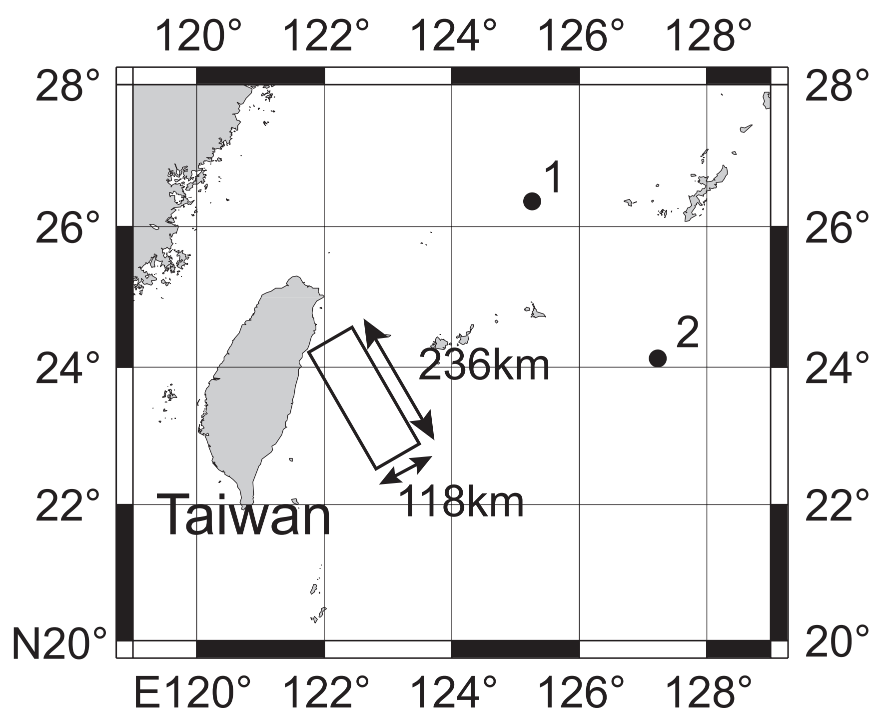

Figure 4: Location of the fault model and two tsunami gauges.

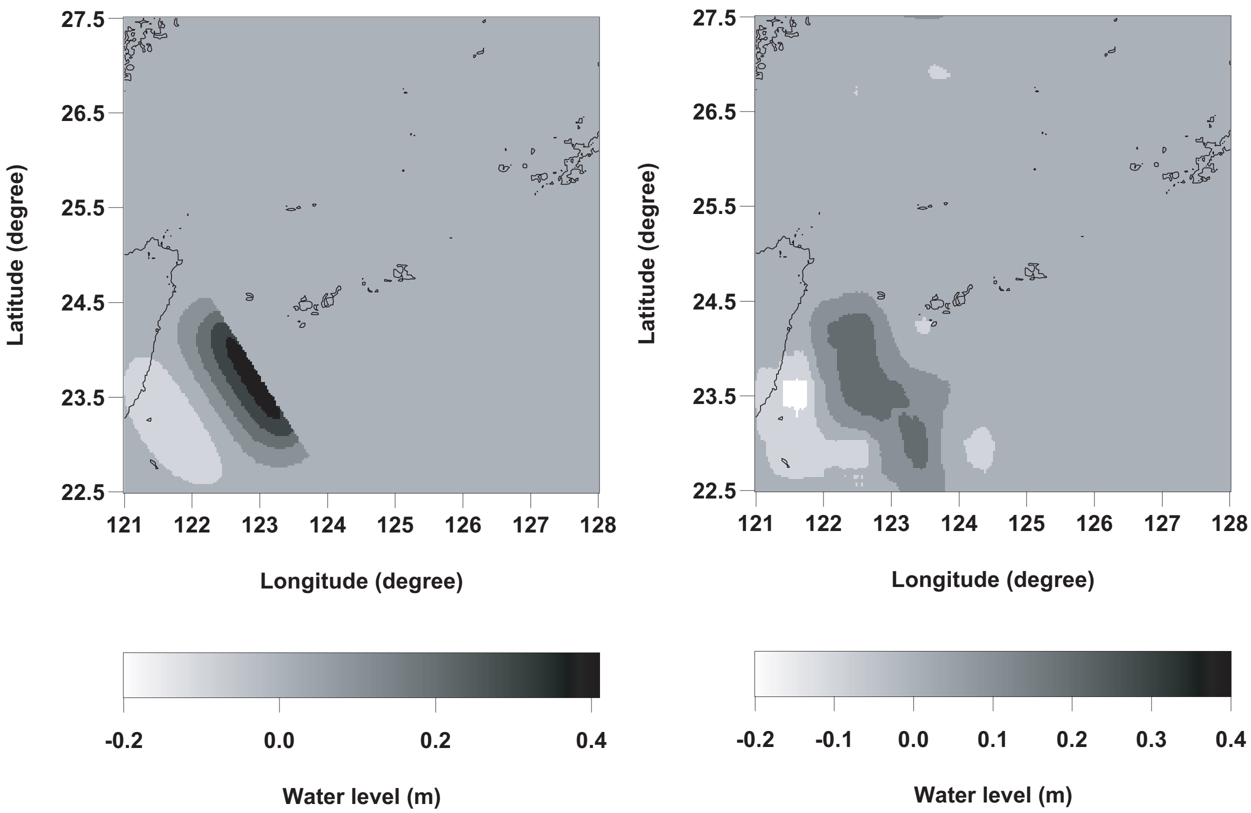

Figure 5: Tsunami initial waveform calculated by the fault model (left) and estimated by the inversion method (right).

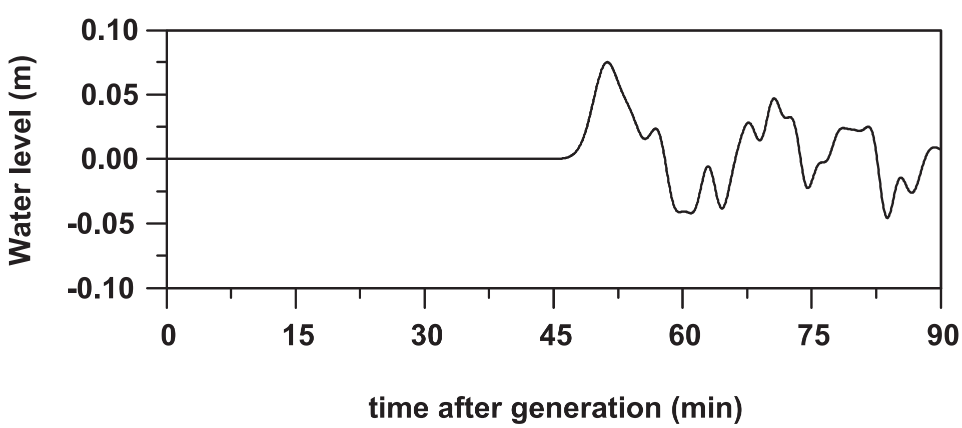

Figure 6: Time-series waveform data at tsunami gauge 1.

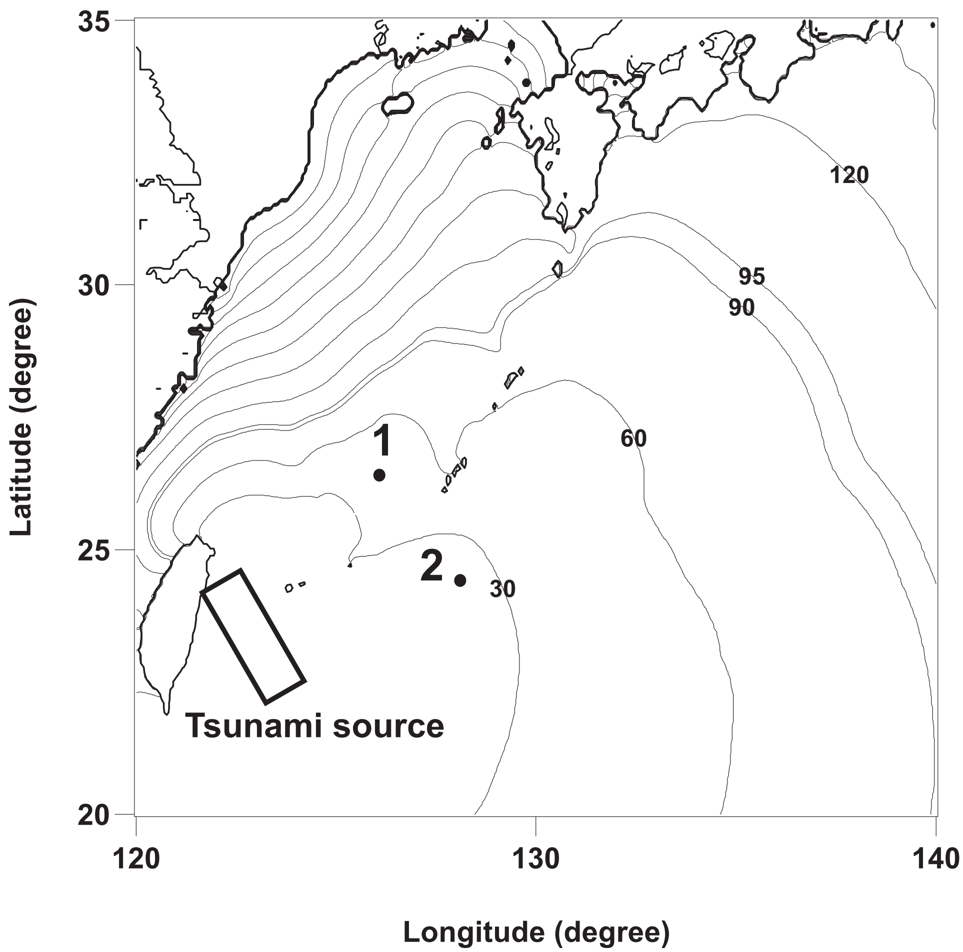

Figure 7: Tsunami arrival time (unit:min). The forecasting time with the inversion method plus the time of their observation and analysis is 50 min.

tsunami heights at the Japanese coast by using obtained initial profile by the inversion method. The sum of these times is about 95 minutes. Therefore at the locations where the tsunami arrival time is more than 95 minutes in Fig. 7, it is possible to give a warning to the residents before the tsunami attacks the coast.

Figure 8 shows the comparison of the tsunami heights forecasted by this system with those calculated by the fault model. The average error was 0.39 m. The average error of the arrival time was 0.007 min.

5. Conclusions

We proposed a new inversion method to estimate a tsunami initial waveform using wavelet transform. This method makes it possible to estimate the location and vertical displacement of water disturbances in the source area. The Beylkin's wavelet is selected to perform the inversion of tsunami

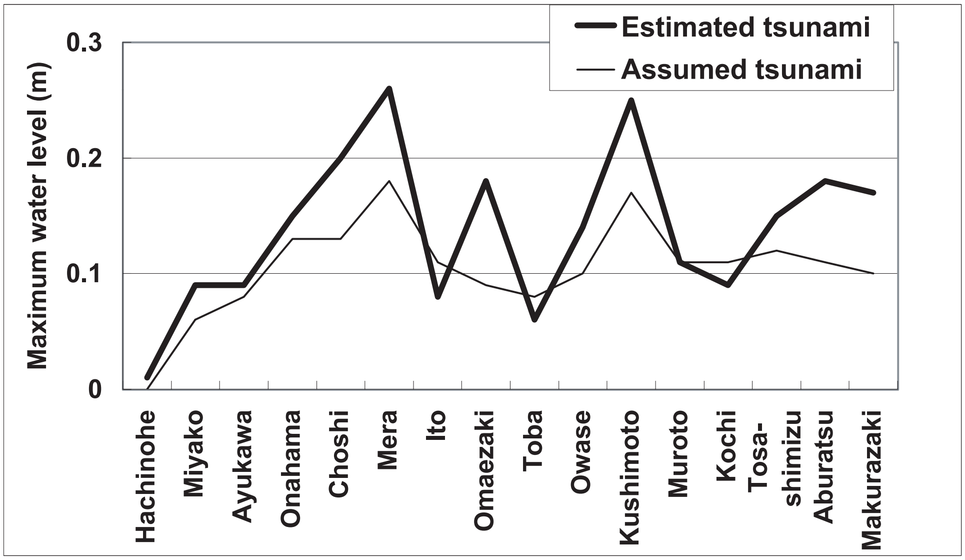

Figure 8: Comparison of the tsunami heights forecasted by the method with those calculated by the fault model.

waveforms recorded at tidal stations. We applied this new inversion method to the 1944 Tonankai Earthquake tsunami, the fault model for which was proposed by Aida (1979). The initial distribution of the water disturbance was estimated using waveforms from four stations. The correlation coefficient for the method was calculated to be 0.87. This value is almost the same as that of the previous methods (e.g., Satake, 1987). However, since the new method does not require the assumption of the location of the tsunami initial waveform, it is possible to estimate the initial distribution of tsunamis caused not only by earthquake but also landslide or volcano.

Moreover, we proposed a real-time tsunami forecast system for the Pacific using the inversion method. In order to check the usability of the system, a numerical simulation was executed assuming an earthquake at sea off Taiwan. It seems clear that this system actually can be used with adequate accuracy.

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